Prof. V. Raghavendra, IIT Tirupati, delivered a talk on 'Peano Axioms' on Sep 14, 2017 at BITS-Pilani, Hyderabad Campus

Abstract

The Peano axioms define the arithmetical properties of natural numbers and provides rigorous foundation for the natural numbers. In particular, the Peano axioms enable an infinite set to be generated by a finite set of symbols and rules.

In mathematical logic, the **Peano axioms**, also known as the **Dedekind–Peano axioms** or the **Peano postulates**, are a set of axioms for the natural numbers presented by the 19th century Italian mathematician Giuseppe Peano. These axioms have been used nearly unchanged in a number of metamathematical investigations, including research into fundamental questions of whether number theory is consistent and complete.

The need to formalize arithmetic was not well appreciated until the work of Hermann Grassmann, who showed in the 1860s that many facts in arithmetic could be derived from more basic facts about the successor operation and induction.[1] In 1881, Charles Sanders Peirce provided an axiomatization of natural-number arithmetic.[2] In 1888, Richard Dedekind proposed another axiomatization of natural-number arithmetic, and in 1889, Peano published a more precisely formulated version of them as a collection of axioms in his book, *The principles of arithmetic presented by a new method* (Latin: *Arithmetices principia, nova methodo exposita*).

The Peano axioms contain three types of statements. The first axiom asserts the existence of at least one member of the set of natural numbers. The next four are general statements about equality; in modern treatments these are often not taken as part of the Peano axioms, but rather as axioms of the "underlying logic".[3] The next three axioms are first-order statements about natural numbers expressing the fundamental properties of the successor operation. The ninth, final axiom is a second order statement of the principle of mathematical induction over the natural numbers. A weaker first-order system called **Peano arithmetic** is obtained by explicitly adding the addition and multiplication operation symbols and replacing the second-order induction axiom with a first-order axiom schema